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FREE CONVECTION AT A VERTICAL PLATE AND IN CLOSED INTERLAYER AT DIFFERENT GAS PRESSURES

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Abstract—Experimental installation for, and results of measuring local heat-transfer coefficients on vertical plates and in closed vertical interlayers at different gas pressures are described.

Based on experimental material treated, criterial relations are suggested which allow calculation of local heat-transfer coefficients from vertical plates within $1 < Ra_x < 10^7$ and local convection coefficients in interlayers at $1 < Ra_m < 10^5$, $0{\cdot}025 < L/H < 0{\cdot}175$.

NOMENCLATURE

x, vertical coordinate [m];
t, temperature [°K];
Δt, temperature drop between interlayer walls [K];
τ, time [s];
C, heat capacity [J/K];
F, heat-transfer surface area of heat flux meter [m²];
q, q_Λ, q_λ, density of total (conduction, radiation, convection), radiative and conductive heat fluxes [W/m²];
Nu, Ra, $= Gr.Pr$, Gr, Pr, Kn, Nusselt, Rayleigh, Grashof, Prandtl and Knudsen numbers;
β, volume expansion coefficient [1/K];
g, acceleration of gravity [m/s²];
ν, kinematic viscosity [m²/s];
η, dynamic viscosity [N.s/m²];
a, thermal diffusivity [m²/s];
λ, thermal conductivity [W/m.K];
ρ, density [kg/m³];
k, adiabatic index;
b, accommodation coefficient;
f, slip parameter;
α, heat-transfer coefficient [W/m².K];
P, gas pressure [torr];
L, thickness of interlayer [m];
H, height of interlayer or plate [m];
v, rate of temperature variation [K/s];
$\bar{\varepsilon}_c, \varepsilon_c$, mean and local convection coefficients.

Subscripts

L, x, characteristic dimension in dimensionless numbers;
$m, 0$, physical parameters to be chosen at arithmetic mean temperature and normal pressure;
a, medium;
t, heat flux meter;
s, surface to be tested.

1. MEASURING TECHNIQUE

LOCAL heat fluxes have been measured with the aid of special heat flux meters whose principle of operation is described in [1], the method in short being presented below.

A heat flux meter consisted of two chrome-plated copper disks 2 and 3, 15 mm in dia. (core) is embedded into plate 1 (Fig. 1a). The disk contains thermocouple 4 serving both for temperature measurement and heating the core. The heat flux meter is fixed at plate 1 with the aid of ebonite spin 5, 1 mm in dia. The core and plate 1 are separated with air space 7; the outer slot is overlapped with flat ring 6. The outer surfaces of the ring, of disk 3 and of plate 1 were polished and chrome-plated. By allowing electric current through thermocouple 4, temperature t_t of the heat flux meter may be increased above temperature t_s of the plate. Current being then switched off, the thermocouple operates as a temperature sensor for the core. The core, when cooled, transfers its heat to plate 1 and into the ambient fluid of lower temperature t_a (Fig. 2). At time τ_0 the temperatures of the heat flux meter core

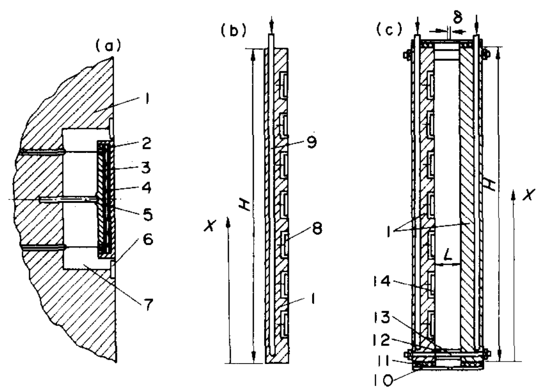

FIG. 1. Diagrams of measuring devices: (a) heat flux meter; (b) single plate; (c) interlayer.

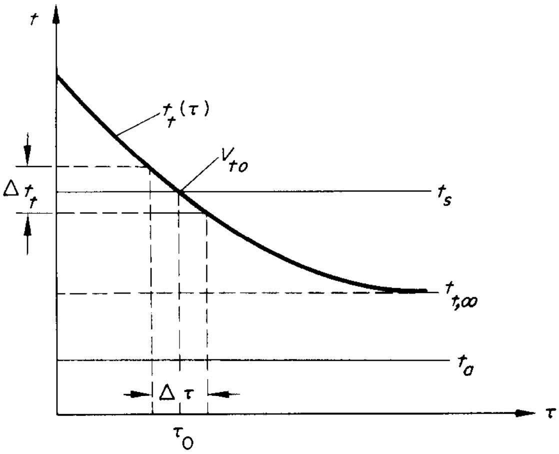

FIG. 2. Heat flux measurement.

and of the plate are equal $t_t(\tau_0) = t_s$ and heat is transferred only to the ambient fluid, the specific heat flux q at this moment being related with the cooling rate v_{t0} of the core, its capacity c_t and surface F by the expression

$$q = -\frac{C_t}{F} V_{t0}, \qquad V_{t0} = \frac{\Delta t_t}{\Delta \tau}. \tag{1}$$

Heat flux meters 8 are placed on vertical isothermal plate 1 (Fig. 1b) whose temperature is kept constant by pumping thermostated liquid through coil 9, being soldered into the plate. The water temperature fluctuations are within $\pm 0{\cdot}1$°K.

Closed air layers formed of isothermal plates 1, 200 × 200 mm (Fig. 1c), present the second aim of our investigation. End slits are covered along the peripheries with lids 10. To reduce the heat transfer, the gap of $\delta = 0{\cdot}1$ mm is left at the splice of the lids; the lids and plates 1 are separated with 3 mm polyfluoroethylene interlayer 11. The interlayer thickness is adjusted by means of thin-wall textolyte spacers 12 put on spins 13 that draw plates 1 together. Seven heat flux meters 14 are set along the height of the hotter plate 25 mm apart. To decrease radiative heat transfer, all internal surfaces of the interlayer are chrome-plated and polished.

A measuring assembly is placed under a glass cap 360 mm in dia. and 500 mm in height for evaporation; the pressure ranges from 10^{-4} to 760 torr. The control measurings show that at atmospheric pressure the error in measured fluxes q between the plates is within 3 per cent.

2. CONVECTIVE HEAT TRANSFER AT RAREFACTION

Consider one of the possible analytic means to account for the effect of rarefaction on convective heat transfer.

We assume that criterial relation of the type

$$Nu = f\left(Ra, \frac{L}{H}\right), \qquad Ra = Gr.Pr$$
$$Nu = \frac{\alpha}{\lambda}.L, \qquad Gr = \frac{\beta g L^3.\Delta t}{\nu^2}, \qquad Pr = \frac{\nu}{a} \tag{2}$$

is known for a continuum gas flow. In these formulas the physical parameters of gas λ, a, $\nu = \eta\rho^{-1}$ depend on pressure. Density is related with pressure P directly

$$\frac{\rho}{\rho_0} = \frac{P}{P_0} \tag{3}$$

while thermal conductivity λ and viscosity η are related indirectly via Kn [2, 3]

$$\lambda = \lambda_0\left[1 + \frac{9_K - 5}{d(k+1)}.\left(\frac{2-b}{b}\right).Kn\right]^{-1},$$
$$\eta = \eta_0\left[1 + \frac{2(2-f)}{df}.Kn\right]^{-1}. \tag{4}$$

Here $d = 1$ and 2 for a flat interlayer and infinite surface; slip factor f is unity [3] for the majority of problems.

It should be remembered that the Knudsen number $Kn = \Lambda/L$ is the ratio of the free molecular path length to some dimension of the surface or interlayer. Since the free path length Λ depends on pressure $\Lambda P = \Lambda_0 P_0$, the relations $\lambda = \lambda(P)$ and $\eta = \eta(P)$ can readily be found in formulae (4).

Following the procedure of [2, 3], we shall find the dependence of the Rayleigh number Ra on pressure. To do this, λ and ν in (2) will be substituted by their values from (4)

$$Nu = \frac{\alpha.L}{\lambda} = \frac{\alpha.L}{\lambda_0}(1 + A.Kn);$$
$$Ra = Ra_0\left(\frac{P}{P_0}\right)^2(1 + A.Kn)(1 + B.Kn); \tag{5}$$
$$A = \frac{(9_K - 5)}{d(k+1)}.\frac{(2-b)}{b}, \qquad B = \frac{2-f}{f}.\frac{2}{d}.$$

The above method involves some assumptions. The consideration is based on equations (2) obtained for a continuum flow. The temperature and velocity drops at the boundaries are allowed for in expressions (4) which are then included into initial equations (2). So the nature of the boundary layer is supposed to remain unchanged which implies that only quantitative (variable thickness, for example) rather than qualitative changes in the boundary layer may occur under rarefaction.

The range of validity of this assumption is still insufficiently verified experimentally [4] and will be discussed below.

3. HEAT TRANSFER OF A SINGLE PLATE

The heat-transfer rate at the height x of a single plate is characterized by the local heat-transfer coefficient α_x and the Nusselt number Nu_x

$$\alpha_x = \frac{q - q_\Lambda}{t_w - t_a}, \quad Nu_x = \frac{\alpha_x . x}{\lambda_f}. \tag{6}$$

At atmospheric pressure the critical relationship is known between Nu and Ra [5–8]

$$Nu_x = M \sqrt[4]{[Ra_x]} \tag{7}$$

where M is a factor that ranges as $0{\cdot}380 \leqslant M \leqslant 0{\cdot}415$ as reported by different authors. Physical parameters included into the dimensionless numbers are chosen at the temperature t_w of the plate.

Using the method described in Section 1 the local heat transfer of a single plate can be measured at different pressures. The numbers Nu and Ra were estimated by formulae (5), the Kn number being so small that $A.Kn \ll 1$ and $B.Kn \leqslant 1$. The Rayleigh number Ra_x estimated by the above method ranged from 1 to 10^7.

The experimental points scatter by no more than 8 per cent against the mean values which can be expressed by the relationship

$$Nu_x = 0{\cdot}4(Ra_x)^{0{\cdot}25}, \quad 1 \leqslant Ra_{x,0}\left(\frac{P}{P_0}\right)^2 \leqslant 10^7. \tag{7'}$$

A negligible difference between measured and predicted values proved that the above method can be used to calculate the heat-transfer coefficient under vacuum and allows the expression of type (5) to be used for description of the measurements in the form

$$Nu_x = Nu_{x0}\left(\frac{P}{P_0}\right)^{0{\cdot}25}. \tag{8}$$

In [7] on the basis of experimental data the relationship

$$\alpha_s = \alpha_{s0}\left(\frac{P}{P_0}\right)^{0{\cdot}25} \tag{8'}$$

has been obtained to calculate the mean surface coefficient of heat transfer from a single plate at reduced ambient pressure.

As follows from (8) and (8′), the local and mean heat-transfer coefficients equally depend on the pressure of the fluid around the plate.

4. CLOSED INTERLAYER WITH VERTICAL ISOTHERMAL WALLS

The measurements were carried out for the following values of the main parameters: thickness of the interlayer $L = 5$, 10, 15, 20 and 25 mm; temperature of the cold plate $T = 298°K = \text{const}$; a temperature drop between the heated and cold plates was $\Delta t = 20$, 40 and 60°K; the air pressure ranged from 10 (pure heat conduction for all interlayer thicknesses) to 760 torr. The dimensionless numbers ranged as $1{\cdot}0 \leqslant Ra_m \leqslant 10^5$; $0{\cdot}025 \leqslant L/H \leqslant 0{\cdot}125$.

The heat-transfer rate in closed interlayers was estimated with the convection coefficient ε_c

$$\varepsilon_c = \frac{q - q_\Lambda}{q_\lambda}. \tag{9}$$

If the interlayer thickness L is chosen as the characteristic dimension for the Nusselt number (2) and the heat-transfer rate is based on the temperature drop Δt between the plates, then the numerical value of the Nusselt number Nu will be equal to the convection coefficient (9).

The analysis of the measured values presented in Figs. 3–5 allows the following conclusions:

(a) The rate of local heat transfer in closed vertical interlayers with isothermal walls may essentially change (almost by one order) along the height of the interlayer.

A behaviour of the local heat-transfer rate is illustrated in Fig. 3. Three main regions may be distinguished along the height of the interlayer. In the lower part of the heated plate the rate of heat transfer is maximal. With increasing the height x the convection

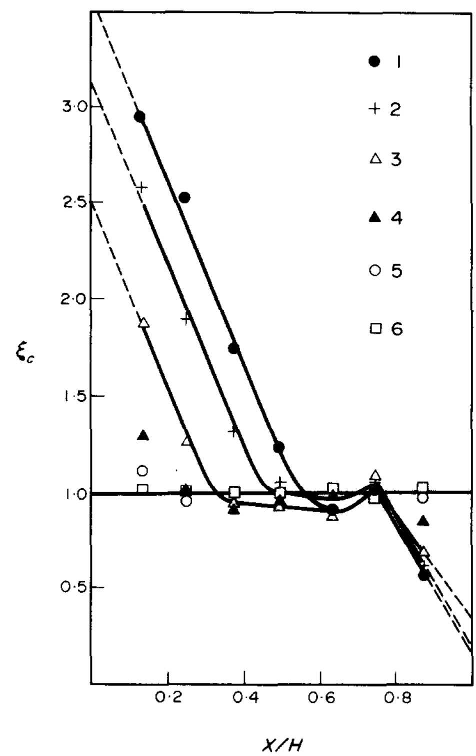

FIG. 3. Plot of ε_c vs X/H for $H/L = 10$ and $\Delta t = 22°K$: 1. 760 torr; 2. 200 torr; 3. 300 torr; 4. 225 torr; 5. 100 torr; 6. 10 torr.

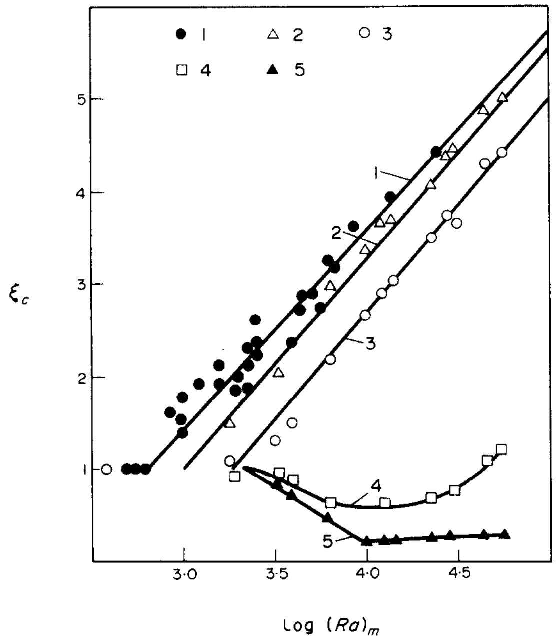

FIG. 4. Limit values of ε_c: 1. $X/H = 0$, $P < 700$ torr; 2. $X/H = 0$, $P = 760$ torr; 3. $X/H = 0{\cdot}125$, $P = 760$ torr; 4. $X/H = 0{\cdot}875$, $P = 760$ torr; 5. $X/H = 1$, $P = 760$.

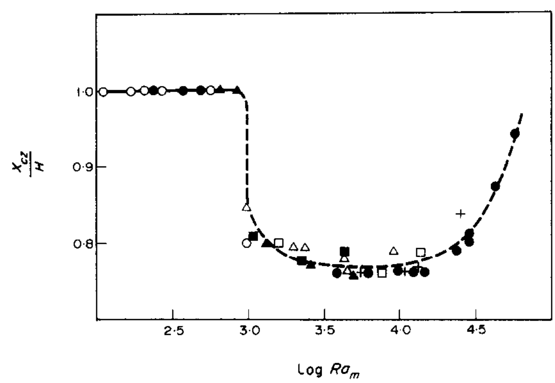

FIG. 5. Region of transition to $\varepsilon_c < 1$. For notations see Fig. 3.

coefficient decreases to unity and remains constant in a certain region. In the upper part of the interlayer the convection coefficient falls again and attains the value of about 0·2.

The existence of the region where the convection coefficient may be less than unity at atmospheric pressure has been found experimentally in [9]. A similar conclusion might be drawn from consideration of numerical solutions of convective heat transfer in interlayers obtained in [10–12].

The investigation of heat transfer at free convection in an interlayer carried out by the present authors confirmed the possibility of existence of the region with $\varepsilon_c < 1$ both at atmospheric and lower gas pressure.

This phenomenon ($\varepsilon_c < 1$) may be readily explained. The upward air flow along the hot plate is due to increase in its temperature. The boundary-layer thickness increases with the height, while the temperature difference between the plate and surrounding gas falls. This results in a decreased heat flux from the surface of the plate to the surrounding gas. In the upper region of the interlayer the temperature difference between the plate and gas may appear to be so small that heat from the heated plate to the cold one is only transferred by radiation. Similar will be the case in the lower region of the interlayer from the side of a cold plate.

In a certain region of the interlayer at free convection the local heat flux may appear to be less than the heat flux in this region only due to gas heat conduction with no convection.

(b) In the interlayer the reduced gas pressure decreases the intensity of gas circulation that involves decrease of local heat fluxes in the region $\varepsilon_c > 1$ and enhances heat transfer in the region $\varepsilon_c < 1$ (Fig. 3).

Beforehand, the integral heat-transfer coefficient over the surface was measured by the method of "additional wall" in the interlayer of the same geometry. It was shown that the results of measurements were correlated by the critical relation of the form [13]

$$\bar{\varepsilon}_c = 1 + \frac{0{\cdot}024 Ra_m^{1{\cdot}4}}{1{\cdot}10^4 + Ra_m} \tag{10}$$

with the RMS deviation within 7 per cent.

The averaging of ε_c by the readings of seven heat flux meters and comparison with (10) showed satisfactory agreement between the results obtained by different methods.

5. CORRELATION OF MEASUREMENTS

For a vertical closed interlayer with isothermal walls the following criterial relations are obtained which relate the convection coefficient (or the Nusselt number) with the Rayleigh number and the height of a local section: at $P = 760$ torr, for the region with $\varepsilon_c \geqslant 1$

$$\varepsilon_c = 2{\cdot}25 \log Ra_{m0} - 4{\cdot}8 \frac{X}{H} - 5{\cdot}75 \tag{11}$$

at $P < 760$ torr, with $\varepsilon_c \geqslant 1$

$$\varepsilon_c = 2{\cdot}1 \log Ra_m - 4{\cdot}8 \frac{X}{H} - 4{\cdot}96. \tag{12}$$

The formulas are valid at $Kn \ll 1$; $1 < Ra_m < 1 \times 10^5$; $0{\cdot}025 < L/H < 0{\cdot}175$.

Under rarefaction Ra_m is related with Ra_{m0} at normal pressure by the relationship

$$Ra_m = Ra_{m0}\left(\frac{P}{P_0}\right)^2.$$

The value of the dimensionless coordinate X_{cr} beginning from which ε_c attains the values less than unity is determined from the plot in Fig. 4.

Tentatively X_{cr} may be found by the following formulas

$$\frac{X_{cr}}{H} = 1 \quad \text{for} \quad 0 \leqslant \log Ra_m < 3$$

$$\frac{X_{cr}}{H} = 0{\cdot}77 \quad \text{for} \quad 3{\cdot}0 < \log Ra_m < 4{\cdot}5.$$

The section of lower heat-transfer rate may take up to 25 per cent of the total height of the interlayer and is disposed at its upper part near the heated plate. Within $X_{cr}/H \leqslant X/H \leqslant 1$ the convection coefficient ε_c may assume the values up to 0·2 (Fig. 5). In the range of $\varepsilon_c < 1$ ε_c can approximately be found by plotting a linear curve of ε_c vs X/H by the two points: $\varepsilon_c = 1$ at $X/H = X_{cr}/H$ (Fig. 5) and $\varepsilon_c = f(Ra)_m$ (curve 5, Fig. 5) at $X/H = 1$.

The comparison of ε_c calculated by formulae (11) and (12) with experimental data showed the following RMS deviations: $\sigma < 7$ per cent at normal pressure and $\sigma < 10$ per cent at $P < 700$ torr.

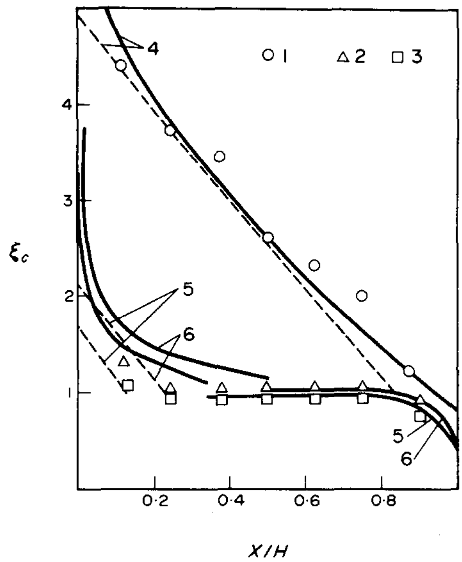

FIG. 7. Relationship $\varepsilon_c = \varepsilon(X/H)$ for vertical interlayers: —, predictions from [9]; ---, predictions by [11]; ○, △, □, experiment; 1, 4, $Ra_{m0} = 58\,300$; 2, 6, $Ra_{m0} = 3100$; 3, 5, $Ra_{m0} = 1930$.

6. THE ANALYSIS OF DATA FROM LITERATURE

In Figs. 6 and 7 the experimental data obtained by the authors are compared with vast reported material. In Fig. 6 solid lines show the results of numerical solution [10] for some particular cases $H/L = 10$; $Gr_L = 7 \times 10^4$; 2.4×10^4; 10^4. The points show the experimental results obtained by the authors at $H/L = 10$, $Gr_L = 4.3 \times 10^4$; 2.2×10^4. Dashed lines present the results of calculation by formulae (11) for the values of Gr_L used in [10]. The results of numerical solution within $1{\cdot}5 < X/L < 10$ agree satisfactorily with experimental data obtained in this work. For small $X/L < 1{\cdot}5$ (corresponding to the lower part of the heated plate) a systematic overestimation of predicted values in contrast to the experiment was observed.

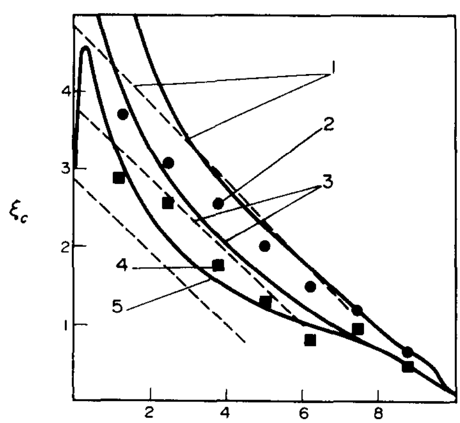

FIG. 6. Relationship $\varepsilon_c = \varepsilon(X/L)$ for vertical interlayers at $H/L = 10$; —, predictions from [10]; ---, predictions by [11]; ●, ■, experiment; 1. $Gr_{m0} = 7 \times 10^4$; 2. $Gr_{m0} = 4.3 \times 10^4$; 3. $Gr_{m0} = 2.4 \times 10^4$; 4. $Gr_{m0} = 2.2 \times 10^4$; 5. $Gr_{m0} = 1 \times 10^4$.

Figure 7 furnishes the results of comparison with the approximate relationships obtained by Eckert and Carlson [9] based on heat-transfer interferograms in closed vertical air layer.

Depending on Gr_L and the parameter H/L in [9] three modes of heat transfer in the interlayer are suggested: heat-conduction region incipient convection at the end regions of the interlayer), transient region and boundary-layer region (fully-developed convection). It should be noted that the authors experimented mainly with transient conditions. For the heat-conduction region only two authors' experiments for $Ra_{m0} = 1930$ and $Ra_{m0} = 3100$ at $H/L = 20$ can be compared with the results of calculation by the formulas of [9]. It was possible to compare the results for the boundary-layer region at $Ra_{m0} = 58\,300$, $H/L = 8$.

As follows from Fig. 7, essential deviations in the results compared are observed only in the initial section at $X/H < 0{\cdot}05$. This can be explained by imperfection of the approximating Eckert–Carlson formulas where at $X \to 0$, $\varepsilon_c \to \infty$. It should be noted that the zone $0 < X/H < 0{\cdot}05$ lies in the region of extrapolation of the present authors experimental results (first heat flux meter has the coordinate $X/H = 0{\cdot}125$). Nevertheless, relationships (11) and (12) may be recommended for estimation of $\varepsilon_c = f(X/H)$ and as the first approximation within $0 < X/H < 0{\cdot}05$.

CONCLUSIONS

1. It is shown that the local convection coefficient may essentially change (almost by one order) along the closed vertical interlayer and assume values less than unity.

2. Relations (11) and (12) are suggested for calculation of the local convection coefficient ε_c for the region $\varepsilon_c \geqslant 1$ at normal and reduced gas pressures in a closed vertical interlayer.

3. Criterial relations (7) and (8) are obtained for calculation of local heat-transfer coefficients of single vertical plates at reduced pressures.

4. The possibility is justified to calculate free-convection heat transfer at reduced pressure by transforming the criterial expressions obtained at normal pressure through substitution of Ra_m in the form of (5).

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CONVECTION NATURELLE SUR UNE PLAQUE VERTICALE ET DANS UNE FENTE POUR DIFFERENTES PRESSIONS DE GAZ

Résumé—On décrit une installation expérimentale et les résultats des mesures locales des coefficients de transfert thermique sur des plaques verticales et dans des fentes verticales fermées, pour différentes pressions de gaz.

Basées sur les résultats expérimentaux, des formules sont proposées qui permettent le calcul des coefficients locaux de transfert thermique sur des plaques verticales pour $1 < Ra_x < 10^7$ et dans les fentes pour $1 < Ra_m < 10^5$, $0{,}025 < L/H < 0{,}175$.

FREIE KONVEKTION AN EINER SENKRECHTEN PLATTE UND IN GESCHLOSSENEN GRENZSCHICHTEN UNTER VERSCHIEDENEN GASDRÜCKEN

Zusammenfassung—Die Ergebnisse der Messung lokaler Wärmeübergangskoeffizienten an senkrechten Platten und in geschlossenen Grenzschichten und verschiedenen Gasdrücken werden dargelegt. Die Versuchseinrichtung wird beschrieben. Ausgehend von behandelten experimentellen Werten werden Kennzahlenbeziehungen vorgeschlagen, welche die Berechnung von lokalen Wärmeübergangskoeffizienten an senkrechten Platten im Bereich $1 < Ra_x < 10^7$ und lokalen Konvektionskoeffizienten in Grenzschichten im Bereich $1 < Ra_m < 10^5$, $0{\cdot}025 < L/H < 0{\cdot}175$ ermöglichen.

ЯВЛЕНИЯ СВОБОДНОЙ КОНВЕКЦИИ НА ПОВЕРХНОСТИ ВЕРТИКАЛЬНОЙ ПЛАСТИНЫ И В ЗАМКНУТОЙ ПРОСЛОЙКЕ ПРИ РАЗЛИЧНЫХ ДАВЛЕНИЯХ ГАЗА

Аннотация — В статье описывается экспериментальная установка и результаты измерений локальных коэффициентов теплообмена на поверхности вертикальной пластины и в замкнутой вертикальной прослойке при различных значениях давления газа. Основываясь на экспериментальном материале данного исследования, авторы предлагают критериальные отношения, позволяющие рассчитать локальные коэффициенты теплообмена от вертикальных пластин при $1 < Ra_x < 10^7$ и коэффициенты локальной конвекции в прослойках при $1 < Ra_m < 10^5$; $0{,}025 < L/H < 0{,}175$.